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Dynamics of a rising and falling cylinder

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Abstract

In the present investigation, we study the dynamics of a cylinder which is allowed to rise or fall freely through a fluid. We find that falling cylinders, oriented horizontally, all descend in a rectilinear path. As one reduces the relative density (or mass ratio, m^*), such that $m^* < 1$ and the cylinders begin to rise, rectilinear trajectories are found to persist. However, when one falls below a special value of the relative density, the body suddenly commences large-amplitude transverse vibration, with horizontal fluctuations of nearly 2 diameters peak-to-peak. It turns out that the critical mass ratio for this phenomenon to appear, $m^*_{\mathrm{crit}} = 0.54$, agrees remarkably well with the critical value for elastically mounted cylinders, defining a similar sudden transition between states. The vigorously vibrating cylinder exhibits a 2P mode of vortex formation, as found previously for elastically mounted bodies in purely transverse motion. This is consistent with the fact that the critical values of relative densities are similar, although the closeness of the values is unexpected when one considers that the freely rising body exhibits streamwise motions. It appears that the existence of a critical relative density for the rising cylinder is a new phenomenon.

1. Introduction

Existing studies of rising and falling bodies have focused primarily on either spherical or spheroidal shapes, such as bubbles [see the review by Magnaudet and Eames (2000)] or solid spherical particles [for example, Jenny et al. (2004)]. In the present investigation, however, we will consider the dynamics of a circular cylinder, in a horizontal orientation, allowed to rise or fall through a fluid under the action of the net buoyancy force (or weight) of the body. For the case of a rising or falling cylinder, studies have been far more limited. Marchildon et al. (1964), and also Stringham et al. (1969) investigated the dynamics of freely falling cylindrical particles, observing primarily pitching vibrations of the cylinder about the midpoint of its span length. These motions are quite different from those we shall discuss here, where our cylinder is restrained to remain horizontal and where there is no motion parallel to the axis.

The dynamics of a rising or falling cylinder are also closely related to the more commonly studied problem of vortex-induced vibration (VIV) of elastically restrained bodies. An overview of various phenomena in VIV may be found, for example, in the recent review of Williamson and Govardhan (2004). For the case of an elastically mounted cylinder with low mass damping, vibrating transverse to the freestream, three distinct branches of amplitude response are found to exist as the normalized velocity, $U^* = U/f_N D$, (where U is the freestream velocity, f_N is the system's natural frequency in water and D is the cylinder diameter) is increased. These branches are the 'Initial' branch, the 'Lower' branch, and a

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further distinctly higher-amplitude mode appearing between these two other branches, namely the 'Upper' branch (Khalak and Williamson, 1999). The different branches are also shown to correspond to different modes of vortex formation. The wake of the initial branch comprises a '2S' mode [using the nomenclature of Williamson and Roshko (1988)], where two single vortices are shed per cycle of oscillation, while the upper and lower branches comprise a '2P' mode, in which two vortex pairs are formed per cycle.

Of particular relevance in recent studies is the discovery of the existence of a critical mass for an elastically mounted cylinder undergoing VIV in a direction transverse to the flow (Govardhan and Williamson, 2000, 2002). It is well known for these systems that the regime of flow speeds, over which synchronized response occurs, becomes wider as the mass ratio is decreased. However, Govardhan and Williamson (2000) found that for the case of a cylinder with a very low mass ratio, $m^* = 0.52$ (below critical mass), that synchronized response persisted beyond the upper speed limits of their water channel facility. Using response data for a variety of cylinders with different mass ratios, they were able to deduce the following equation for the frequency in the lower branch:

$$f^*_{\text{lower}} = \sqrt{\frac{(m^*+1)}{(m^*-0.54)}}, \tag{1}$$

where $f^* = f/f_N$, with f the oscillation frequency and f_N the natural frequency. This frequency equation suggests the fact that for mass ratios below a value of 0.54, referred to as the critical mass ratio, the lower branch of response cannot be reached and will not exist. In its place, large-amplitude synchronized vibrations were predicted to exist for an infinitely wide range of normalized flow velocities, according to

$$U^*_{\text{end of synch}} \approx 9.25\sqrt{\frac{(m^*+1)}{(m^*-0.54)}}. \tag{2}$$

Subsequently, Govardhan and Williamson (2002) developed an experiment that allowed them to determine the dynamics of a cylinder at an infinitely large normalized velocity. To achieve this, the natural frequency was set to zero by removing the springs that provided the restoring force to the system, resulting in an infinite U^*. By gradually removing mass from the system, they showed that when the mass ratio exceeds a critical value, $m^*_{\text{crit}} = 0.542$, that the cylinder exhibits negligible vibration, but upon reducing the mass ratio below this value, the cylinder would suddenly begin to oscillate vigorously. A similar phenomenon of a critical mass plays a key role in the dynamics of rising and falling cylinders.

Since the vortex dynamics for a freely rising or falling cylinder are coupled with the cylinder motion, a change in the character of the vortex wake can significantly affect the body vibration, and vice versa. In the case of two-degree-of-freedom controlled oscillations, Jeon and Gharib (2001) showed that streamwise motion can, under certain conditions, suppress formation of the secondary vortex in what would otherwise be a 2P mode, replacing it instead with a 2S mode. Jauvtis and Williamson (2004) studied an elastically mounted cylinder free to oscillate in both transverse and streamwise directions. Unlike the forced vibrations of Jeon and Gharib; such free vibration experiments only admit the subset of motions yielding positive energy transfer from fluid to body motion. They found that for low enough mass ratios, $m^* < 6$, there exists a 'super-upper' branch of response in which the cylinder sheds two triplets of vortices per cycle of oscillation [defined as a 2T mode, in the nomenclature of Williamson and Roshko (1988)]. This mode led to very large amplitudes up to 3 diameters peak-to-peak.

2. Experimental details

Our experiments were performed in a vertical tank with dimensions $0.4 \times 0.4 \times 1.5\,\text{m}^3$, filled with water with a kinematic viscosity, ν, of $0.95 \times 10^{-6} \pm 0.02 \times 10^{-6}\,\text{m}^2/\text{s}$. Inside the tank were a set of vertical false transparent walls, very carefully aligned to be parallel to each other. The horizontal cylinder (with diameter, $D = 1.91\,\text{cm}$, and length, $L = 35.3\,\text{cm}$) spanned the full width between the false walls (less a small gap on the order of $10^{-3}\,\text{m}$ on either side) to prevent translation along its axis. This allowed for motion in the direction of its rise (X) and also in a direction transverse to the mean rising velocity (Y), but without pitching or motion in the direction of the cylinder axis. The gap did not affect the cylinder dynamics, provided that the cylinder was launched carefully, as described below, to prevent rubbing against the false walls.

Buoyant cylinders were held in place at the bottom of the tank using a combined system of a hook and electromagnets. The hook would hold the cylinder, after its insertion in the tank, while the water settled down. Shortly prior to running an experiment, the hook was removed, and the electromagnets were used to hold the cylinder before its release. This allowed for a remarkably smooth launch of the body, in preference to the hook itself. The magnets could not be used for longer durations, as they heated up, causing convection currents. For falling cylinders, this apparatus could be inverted and placed at the top of the tank. The range of Reynolds numbers for these experiments was between 3800 and 7000. The cylinders used in this study were hollow and empty of water, allowing for their mass ratios to be altered by adding or removing a number of small weights within the interior of the cylinder. For each cylinder, the added weight was distributed symmetrically around the cylinder's longitudinal axis, and symmetrically with respect to the span center. After adding weight, the cylinder was sealed with flat-faced, watertight endcaps.

For the purpose of employing Digital Particle Image Velocimetry (DPIV), the flow was seeded with 14-μm silver-coated glass spheres, which were illuminated by a sheet of laser light from a 5 W continuous argon ion laser. Pairs of particle images were acquired using a high-resolution CCD Kodak Megaplus (1008 × 1018 pixels), and analyzed using cross-correlation of sub-images. The viewing area of the camera was $25.5 \times 25.8\,\text{cm}^2$, with corresponding time between images being 10 ms. The resulting vorticity fields were phase averaged over approximately 10 cycles using the cylinder position as a reference. Further details pertaining to the implementation of the cross-correlation technique and to the level of particle seeding may be found in Govardhan and Williamson (2000), where the same arrangement of DPIV was implemented.

3. The existence of a critical mass for a rising and falling cylinder

We first performed experiments using a falling cylinder ($m^* > 1$). These trajectories showed negligible transverse motion, with the cylinder falling in a vertical, straight path. By removing weight from the cylinder, we reduced

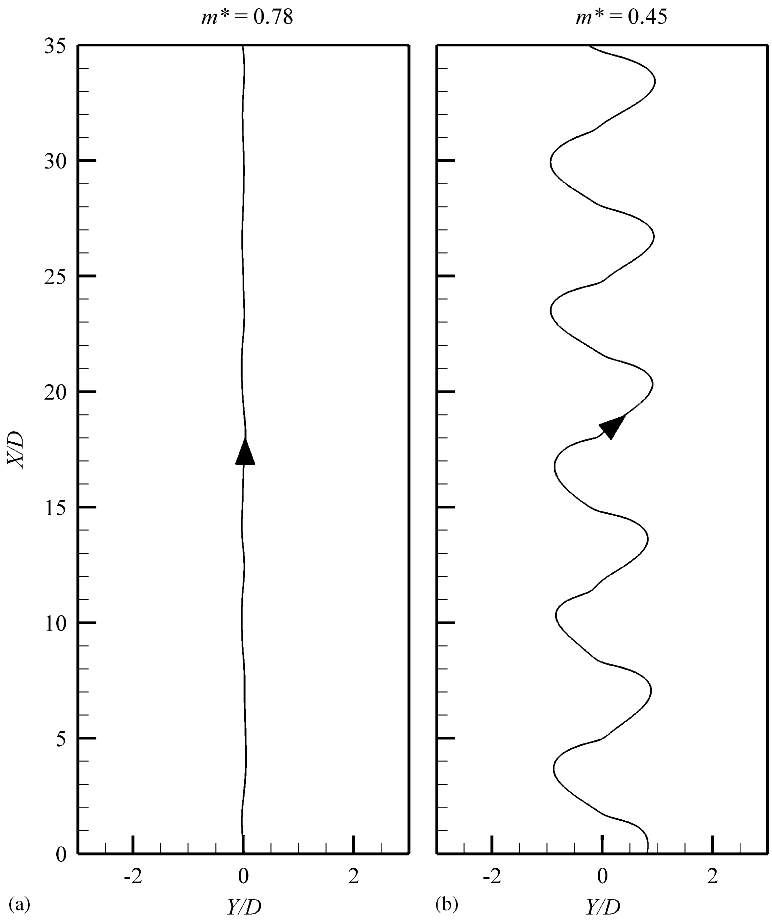

Fig. 1. (a) A buoyant cylinder ($m^* = 0.78$) rises to the water surface ($X/D = 35$) with negligible transverse motion. (b) A slightly lighter cylinder ($m^* = 0.45$) exhibits significant vibrations with a peak-to-peak amplitude of two diameters. The *same cylinder* was used for both experiments, with only small weights being removed in (b) to reduce the mass. Reynolds numbers are: (a) Re ≈ 5000; (b) Re ≈ 3800.

its mass ratio so that the cylinder became buoyant ($m^* < 1$); but, despite being less dense than the surrounding fluid, the cylinder continued to travel with a straight trajectory, in this case, vertically upwards. Further reductions in mass ratio appeared to affect the dynamics of the cylinder very little. However, at a certain point, a further reduction in mass resulted in a catastrophic change in the motion of the cylinder. Rather than rising rectilinearly, the cylinder would now oscillate with an amplitude of around 2 diameters peak-to-peak ($A_Y^* =$ mean amplitude/diameter ≈ 1). The striking difference between these two cases is shown in Fig. 1. It should be noted that the motion of the vibrating cylinder in Fig. 1(b) is highly periodic, with even the small 'kink' near the centerline of the trajectory being repeatable cycle-to-cycle. For the case of a cylinder just slightly heavier than the critical mass ratio, an initial transient could be induced by perturbing the cylinder as it was launched. This transient was damped out rapidly, after which its trajectory resembled that of Fig. 1(a), suggesting that even for a mass ratio barely above the critical value, the cylinder was disinclined to vibrate. No overlap of the two branches of response in Fig. 2 was observed.

The character of this change in the motion of the rising or falling cylinder may also be seen in Fig. 2, where a reduction in mass below a certain critical value, m^*_{crit}, found here to be 0.54, leads to a large jump in the amplitude of vibration. As mentioned earlier, such a phenomenon was also discovered by Govardhan and Williamson (2000, 2002), who performed experiments on a cylinder mounted on an air bearing in a water channel. Using elastically mounted cylinders, they found a critical mass ratio, $m^*_{\text{crit}} = 0.54$, while their experiments using a cylinder with no attached springs yielded $m^*_{\text{crit}} = 0.542$. Despite the distinctly different experimental systems between the freely rising body and the bearing-mounted bodies, it is fascinating to discover a remarkable similarity in the value of the relative density marking the transition between system states.

It is worth mentioning that we detected a very small rotational displacement of the cylinder about its axis, around 5°, when the body vibrates (for $m^* < 0.54$). The fact that the same cylinder is used throughout this study, with careful axisymmetric placement of weight within the body, and the fact that a closely related critical mass is also found for the air-bearing mounted bodies of Govardhan and Williamson (2000, 2002), suggests that such small rotational displacement is an incidental effect, and is not relevant to the existence of the phenomena presented in this study.

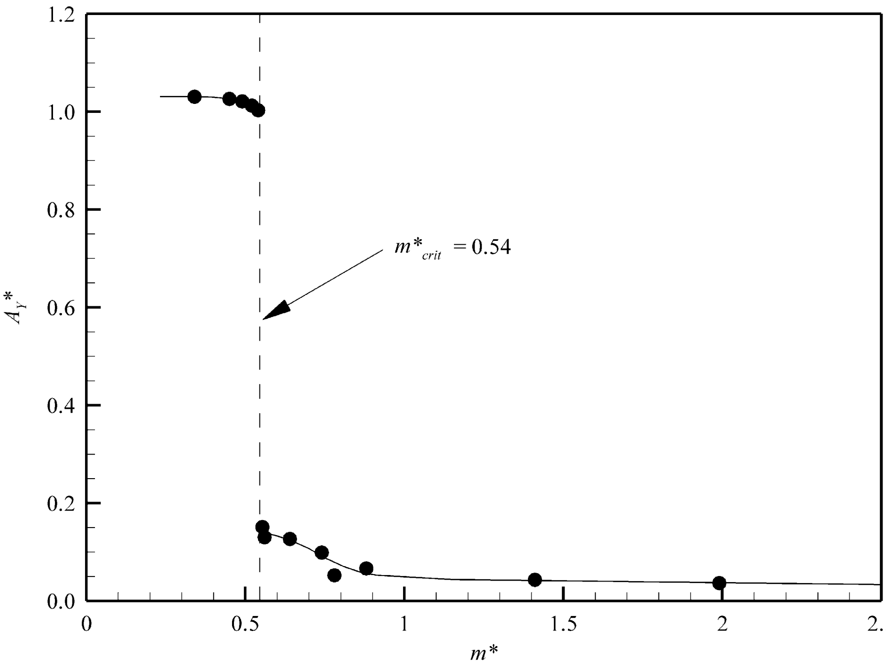

Fig. 2. As the mass ratio, m^*, of the freely rising cylinder is reduced below a critical value, m^*_{crit}, there occurs a sudden jump in amplitude of response, A_Y^*. We find a critical mass, $m^*_{\text{crit}} = 0.54$, in excellent agreement with the experiments of Govardhan and Williamson (2002), who obtained $m^*_{\text{crit}} = 0.542$ using a cylinder mounted on air bearings.

4. Vortex dynamics and the operating point

Since the vortex dynamics of a vibrating cylinder have a significant effect on its response, we measured vorticity in the wake of bodies that rose with essentially no vibration, and also those exhibiting large amplitudes of oscillation. The cylinders that rose with a straight trajectory generated a 2S mode of vortex formation, as one might expect, shown in Fig. 3. Interestingly, despite the presence of strong coherent vortices, which indubitably give rise to a significant lift force, the motion of the *completely unconstrained* cylinder was not affected, essentially rising with negligible transverse motion, as seen for example in Fig. 1(a).

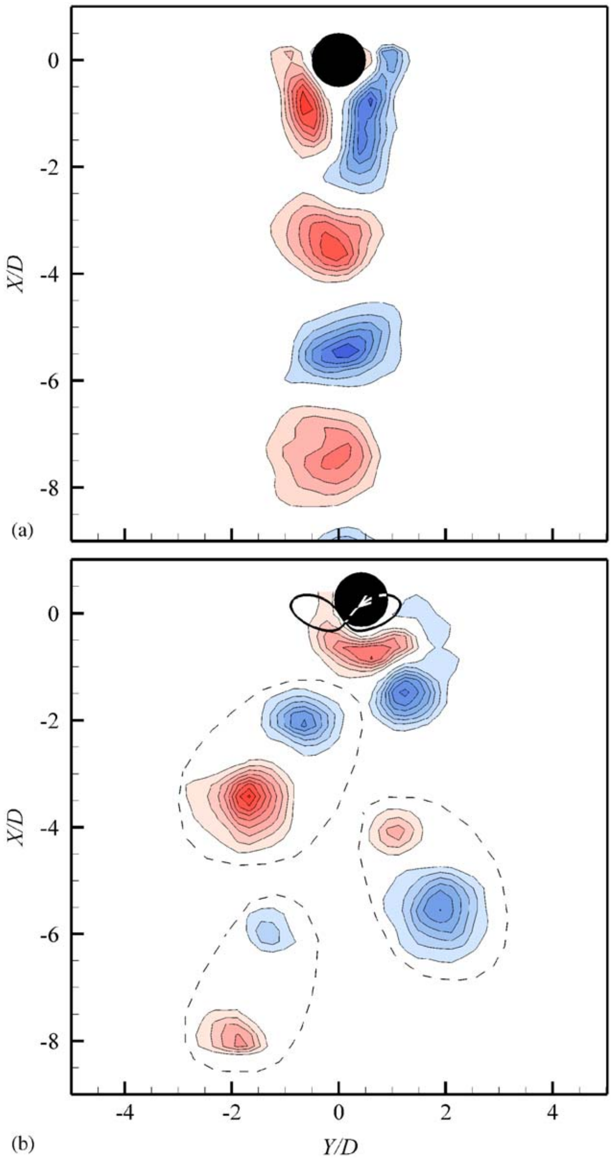

Fig. 3. The trajectories of the rising cylinders shown earlier in Fig. 1 are associated with two different modes of vortex formation shown here. (a) The cylinder rising with no oscillation ($m^* = 0.78$) exhibits a 2S mode, while (b) a lighter ($m^* = 0.45$) cylinder, vibrating with a large amplitude, shows a 2P mode of vortex formation. The dashed lines in (b) are introduced to illustrate the counter-rotating vortex pairs of the 2P mode. Vorticity contour levels are $\omega D/U = \{\pm 0.2, \pm 0.4, \pm 0.6, \ldots\}$ in (a), and $\omega D/U = \{\pm 0.4, \pm 0.8, \pm 1.2, \ldots\}$ in (b).

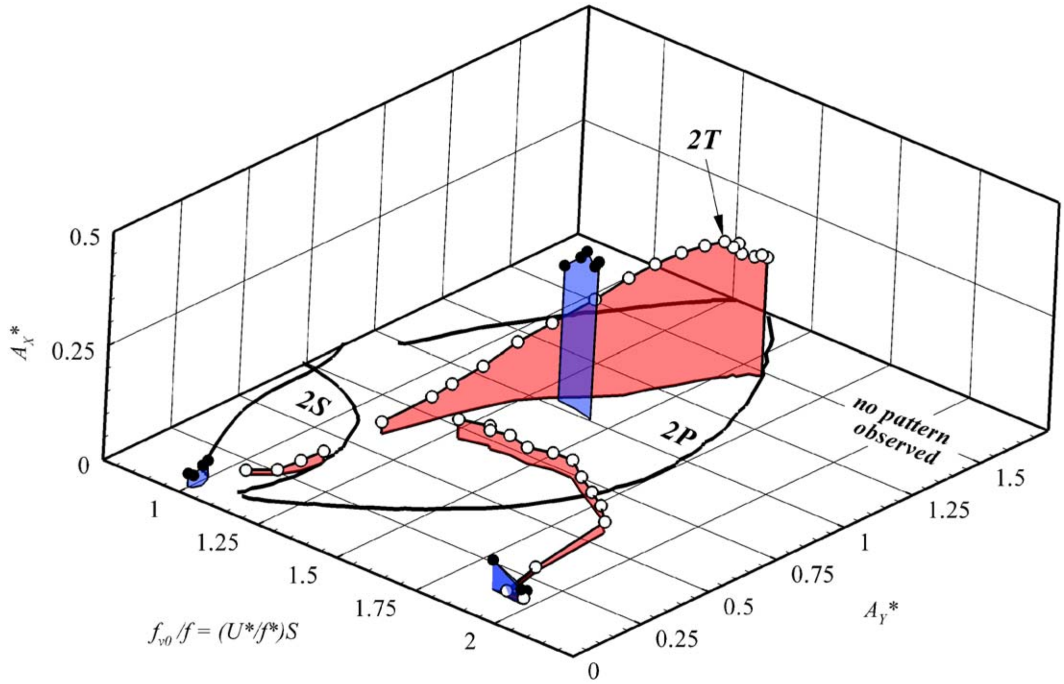

Fig. 4. The 'operating point' of a freely rising cylinder (blue) in the three-dimensional amplitude–frequency plot occurs in a different space from the super-upper branch of response for a two-degree of freedom elastically mounted cylinder (red). This is consistent with the fact that the vortex formation modes are quite distinct; the rising cylinder exhibits a 2P mode of vortex formation, while the elastically mounted case has a 2 T mode. ●, Freely rising cylinder; ○, $X - Y$ motion of an elastically mounted cylinder (Jauvtis and Williamson, 2004); —, Williamson and Roshko (1988) map of wake modes, defined for $A_X^* = 0$.

It has been shown by Jauvtis and Williamson (2004) that streamwise motion may influence the mode of vortex shedding behind an elastically mounted cylinder in two degrees of freedom, most noticeably by the existence of the 2T mode comprising two triplets of vortices shed per cycle. One may introduce the concept of an 'operating point' in the plane of (amplitude-frequency), or (A_Y^* versus f_{v0}/f), where the system resides when undergoing large-amplitude vibration for $m^* < 0.54$. (Here we define f_{v0} as the frequency of vortex shedding for a nonoscillating body, while f is the body oscillation frequency.) This concept was introduced by Govardhan and Williamson (2002) for the springless cylinder mounted on air bearings. In the present case, our 'operating point' will include streamwise motion such that the motion is defined by $\{A_X^*, A_Y^*, f_{v0}/f\}$ in a three-dimensional form of the Williamson and Roshko (1988) map of modes (where $A_X^* = 0$). Observing our operating point (the blue column of Fig. 4), one might be tempted to infer from its close proximity to the super-upper branch (the red region) that a 2 T mode might be expected to form behind the rising cylinder. Instead, by showing the location of the operating point in this three-dimensional space that includes the streamwise amplitude, A_X^*, it becomes clear that our operating point (blue column) lies distinctly above the super-upper branch.

In place of the 2T mode, it is interesting that the freely rising cylinders, which exhibit vibrations both transverse and streamwise to the incident relative flow, yield a 2P vortex formation mode. This is the mode found typically in the case of purely transverse motion of elastically mounted or controlled bodies (in the lower plane of Fig. 4, where $A_X^* = 0$). Though it is not obvious from the trajectory in Fig. 1(b), the light rising cylinders did vibrate in the streamwise direction with an amplitude, $A_X^* \approx 0.3$. These results suggest the intriguing notion that there may exist other three-dimensional surfaces or volumes in the $\{A_X^*, A_Y^*, f_{v0}/f\}$ space corresponding to different vortex formation modes for freely vibrating or elastically mounted bodies.

5. Conclusions

In this study, we find that rising and falling cylinders follow a straight trajectory, unless the relative density is reduced below a critical value, when the body suddenly commences large-amplitude transverse vibration, with horizontal fluctuations of nearly 2 diameters peak-to-peak. In essence, the system dynamics can change catastrophically from straight trajectories to large-amplitude oscillations, simply as a result of a very small reduction in the mass of the rising body. It turns out that the critical mass ratio, $m^*_{crit} = 0.54$, agrees remarkably well with such a critical value found in the case of elastically mounted cylinders, which also marks the sudden transition between distinct states of the system (Govardhan and Williamson, 2000, 2002).

Vorticity measurements indicate that the wake behind the freely rising body in rectilinear motion comprises the 2S mode or classical von Karman street, which is not unexpected. The vigorously vibrating cylinder exhibits a 2P mode of vortex formation. Interestingly, this is quite unlike the 2T wake mode for a two-degree-of-freedom cylinder restrained by springs (Jauvtis and Williamson, 2004). However, the similarities in the vortex dynamics with the purely transverse oscillations of an elastically mounted cylinder are consistent with the closely similar critical mass observed in the two quite different systems. Nonetheless, it is surprising and perhaps coincidental that the values of the critical mass ratio found for these experiments are nearly identical, because one might reasonably expect that the critical mass of a cylinder *freely rising* through a fluid, in two degrees of freedom, would be different from the one for a body *restrained* to move transverse to the flow, in one degree of freedom, in a water channel.

The observation of a critical relative density for rising or falling cylinders is a new phenomenon and one might suspect its existence for other free bodies in a fluid. Further research is underway to both understand the physical mechanism underlying the existence of the critical relative density in these problems, and to study the free rise or fall of other bodies, such as spheres.

Acknowledgements

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